Received 22 December 2021
Accepted 25 January 2022

DOI: 10.52547/CMCMA.1.1.48
AMS Subject Classification: 65D18; 68U10; 90C08

Comparing image segmentation methods using data envelopment analysis

Hassan Bozorgmanesh[a]

In this paper, a model based on data envelopment analysis is used for comparing different image segmentation methods and also for the purpose of finding the best parameter among certain values for a method. The criteria for choosing inputs and outputs are explained and in the end, some examples are presented to demonstrate how this model works.

Keywords: data envelopment analysis (DEA); image segmentation; efficiency.

1. Introduction

Image segmentation is the process of dividing an image into meaningful parts, while this definition can be ambiguous, it more or less states what image segmentation is all about. Image segmentation as an integral part of image processing has many applications, examples include medical sciences [1], geoinformatics [2], biomechanics [3], Multimedia [4] and etc.

Many image segmentation methods can be found in the literature. It is important to know which one of these methods is more appropriate and efficient for a specific class of images. Another important information is recognizing what value is suitable for a parameter that appears in a segmentation method, choosing an inappropriate value for a parameter can lead to an imprecise segmentation.

Whenever the concept of evaluating performance appears, data envelopment analysis (DEA) can be used as an useful and strong instrument for obtaining it. DEA was and still is being used for evaluating efficiency for a wide variety of applications, like economy, management, engineering and etc (see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]).

DEA is based on the input-output context and it is a flexible system which can be easily understood and solved. Here, DEA is used for proposing a model which can be applied for comparing different image segmentation methods and also choosing a suitable value for a parameter of any segmentation method.

The work of this paper describes a way to handle high number of inputs and outputs, that is, instead of using each image for a separate input and output, the average of computation times and accuracies of different classes of image (animals, buildings, etc.) are used as inputs and outputs.

2. Main Method

In this section, first a survey of two image segmentation methods is given and then DEA is introduced briefly. After that, the main model is described for calculating efficiency. In the end of section, some examples are presented to show how the model works in practice.

2.1. Image Segmentation Methods

Usually, the result of an image segmentation method is a set of boundaries that separates important and meaningful regions from each other. Here, three methods which are used in this paper are introduced briefly.

[a] *Department of Applied Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran. Email: h_bozorgmanesh@sbu.ac.ir.*
[*] *Correspondence to: H. Bozorgmanesh.*

2.1.1. Chan - Vese This method is a region-based method and it uses a simplified version of Mumford-shah functional. Chan-Vese method achieves the boundaries of an image by minimizing the following functional:

$$F(c_1, c_2, \phi) = \mu \int_{\Omega} |\nabla(H(\phi(x)))|\, dx + \int_{\Omega} |I(x) - c_1|^2 H(\phi(x)) dx + \int_{\Omega} |I(x) - c_2|^2 (1 - H(\phi(x))) dx \tag{1}$$

in which ϕ is a signed distance function to the boundary, H is the Heaviside function, c_1 and c_2 are two constants that represent the mean pixel intensity inside and outside the ϕ (boundary) respectively and μ is the regularization parameter. By assuming that ϕ is fixed, we can obtain c_1 and c_2 as

$$c_1 = \frac{\int_{\Omega} I(x) H(\phi(x)) dx}{\int_{\Omega} H(\phi(x)) dx}, \quad c_2 = \frac{\int_{\Omega} I(x)(1 - H(\phi(x))) dx}{\int_{\Omega} (1 - H(\phi(x))) dx}, \tag{2}$$

and Euler-Lagrange equation of Functional (1) is as following

$$\frac{\partial \phi}{\partial t} = \delta(\phi(x)) \left(\mu\kappa - |I(x) - c_1|^2 + |I(x) - c_2|^2\right). \tag{3}$$

By using (2) and (3), Chan-Vese method obtains boundaries. For more information about this method refer to [17].

2.1.2. Bernard This method is considered a region-based method and is newer than the previous ones. In this method, the following functional is minimized:

$$\int_{\Omega} \left(H(\phi(x))\,(I(x) - c_1)^2 + (1 - H(\phi(x)))\,(I(x) - c_2)^2\right) dx. \tag{4}$$

Here, we consider ϕ as a linear combination of B-spline basis functions:

$$\phi(x) = \sum_{k \in \mathbb{Z}^d} c\,[k]\, \beta^n \left(\frac{x}{h} - k\right) \tag{5}$$

in which $\beta^n(.)$ is the uniform symmetric n-degree B-spline. $c\,[k]$ is the coefficients of B-spline representation and h is a parameter that decides the level of smoothness of contour (zero level set of ϕ). For more information, see [18].

2.2. DEA

As it is stated before, DEA is based upon input-output relations. It is a method for comparing different entities, which are called Decision Making Units (DMUs). Here, DMUs are different image segmentation methods which are compared to each other. Every DMU has a set of inputs and outputs. For example, a hospital which has n doctors and m nurses (two inputs) and k patients (one output) are cured there in a month, is a DMU and it can be compared to other hospitals with the same set of inputs (nurses and doctors) and outputs (cured patients).

For every DMU, an optimization problem is created and solved in order to obtain the efficiency of this DMU in comparison to other DMUs. It can be said that this optimization problem obtains the best weights for inputs and outputs so that the DMU which its efficiency is being calculated, has the best performance. To put it simple, if in this best situation, there are other DMUs that perform better than current DMU, then it can be said that this DMU is inefficient.

Suppose there are n DMUs, m inputs and s outputs. The efficiency of DMU$_o$ (o-th DMU) is measured by the following optimization problem:

$$\begin{aligned} &\max_{v,u}\ \theta = \tfrac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo}} \\ &s.t. \begin{cases} \tfrac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \le 1 \ (j = 1, \dots, n) \\ v_1, v_2, \dots, v_m \ge 0 \\ u_1, u_2, \dots, u_s \ge 0 \end{cases} \end{aligned} \tag{6}$$

In which v_i and u_r are weights for the i-th input and r-th output respectively, $(x_{1j}, x_{2j}, \dots, x_{mj})$ and $(y_{1j}, y_{2j}, \dots, y_{sj})$ are input and output data for j-th DMU.

In this model, the objective function measures $\frac{output}{input}$ for DMU$_o$ and the model limits the amount of this fraction to be less or equal to 1, the objective function measures the efficiency since we want to have more output and less input. Model 6 is not a linear problem. Linear problems are easier to solve because there are powerful algorithms for obtaining their solutions. Therefore, Model 6 is converted to the following optimization problem (for details refer to [19]):

$$\begin{aligned} &\max_{v,z}\ \theta = u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so} \\ &s.t. \begin{cases} v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo} = 1 \\ u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj} \le v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj} \quad (j = 1, \dots, n) \\ v_1, v_2, \dots, v_m \ge 0 \\ u_1, u_2, \dots, u_s \ge 0 \end{cases} \end{aligned} \tag{7}$$

If $\theta = 1$, then DMU$_o$ is called efficient, otherwise it is inefficient. Model 7 is called a CCR model.

Definition 1 *If a DMU$_o$ is inefficient then* $R_o := \left\{ j : \sum_{r=1}^{s} u_r^* y_{rj} = \sum_{i=1}^{m} v_i^* x_{ij} \right\}$ *is called the reference set of DMU$_o$. This set of indexes represents those DMUs that caused DMU$_o$ to be inefficient.*

Remark 1 *A difference of DEA with statistical methods lies in the fact that the DEA provides the best situation for a DMU and see if it can be efficient in that environment in comparison to other DMUs, while statistical methods (like least squares regression) consider the average performance of a variable.*

2.3. Main Model

In this paper, the proposed DEA-based model is designed for two purposes. First, finding an optimal parameter for an image segmentation method. Second, comparing a group of image segmentation methods. The efficiency of every method is obtained and then this score is compared to the scores of other methods. The computation time that an image segmentation algorithm consumes is considered as an input. For the output, there is a need to have a criterion of how "accurate" a segmentation is, this accuracy can play the role of an output for the DEA model. For obtaining this accuracy, a predetermined segmentation of images is used in order to compare it to the segmentation of a algorithm. Therefore, the pictures and segmentations of BSDS500 [20] database which includes a wide variety of images and their "human" (or perfect) segmentations, are used.

For the proposed model, a fixed number of images is chosen and then image segmentation methods are performed on these images and we see how much time a method consumed on every image (input) and how accurate it is (output). If a large number of images is chosen, that means there are many inputs and outputs. In this case, a desirable result cannot be obtained [22, p. 211] because the number of inputs and outputs is needed to be not too much higher than the number of DEAs (otherwise it would turn out that almost every method is efficient). For circumventing this problem, categories or classes of images are used instead of images, that is, the images are first divided into some categories or clusters (for instance animals, buildings and etc) and algorithms are run on them, after that the average or a different notation of cluster center is used for representing that category, then the cluster center can be chosen as an input and an output for the DEA model.

For a better understanding of the proposed model, it is illustrated by examples.

Figure 1. Ten pictures that is included in the human face category from BSDS500 database [20].

Remark 2 *For running different algorithms in this paper, Creaseg software [21] which contains six segmentation algorithms, is used. Creaseg software is a useful tool for comparing and obtaining the computation time of an algorithm for any image. It also contains different measures of accuracy to compute the preciseness of an image segmentation with respect to the reference segmentation. In this paper, the inverse of Mean Sum of Square Distance (MSSD) is utilized for the output, using the inverse of MSSD means that a number with higher value suggests more accuracy and zero indicates that there is no match at all, thus it can be used as an output for the DEA model. (for more information about MSSD and Creaseg software refer to [21]).*

Remark 3 *It is obvious that the computation time of an algorithm depends on how well it has been written. Thus, if we want to obtain meaningful results, all algorithms should have written in their fastest form as much as possible, otherwise we may encounter different results with the same segmentation methods.*

Example 1 *Considering Chan-Vese method, as it stated in Section 2.1.1, the user must assign a regularization parameter μ for this segmentation method. Suppose we want to choose one of 0.01, 0.2, 0.4, 0.6, 0.8 or 1 for μ such that Chan-Vese method gives the best result. There exist 50 images that are divided it into five categories: animals, buildings, human faces, nature and objects (see Figure 1). After running Chan-Vese algorithm on images of each category, the average of the segmentation computation times (input) and accuracies of image segmentations (output) for every category are computed.*

Name of image in BSDS500	Time with $\mu = 0.01$	MSSD with $\mu = 0.01$
6046	51.924s	1753.111
8068	37.587s	121.469
41006	28.407s	1047.113
41029	35.442s	940.165
41096	37.748s	1293.872
42049	60.539s	417.753
43051	47.975s	4208.296
66075	60.258s	1089.826
87015	36.964s	2374.000
306052	36.066s	1378.868

Table 1. The category of animal images with the results obtained from Chan-Vese model with $\mu = 0.01$.

We see the result of running Chan-Vese algorithm on the animal category images in Table 1. Now, the average of times 43.2910s is an input and after dividing values of MSSD by 1000 (this is done to avoid losing meaningful numbers) and inversing it, averaging gives 1.6290 as an output for DEA with $\mu = 0.01$. So, we achieved one input and one output for the DEA $\mu = 0.01$, we repeat this process for every μ and obtain one input and output for them. Then we do this for another category and in total, there are five inputs and five outputs for the implementation of the model (see Table 2 and 3). Therefore overall, model is as follows:

$$\max_{v,z} \theta = u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{5o}$$
$$s.t. \begin{cases} v_1 x_{1o} + v_2 x_{2o} + \cdots + v_5 x_{5o} = 1 \\ u_1 y_{1j} + u_2 y_{2j} + \cdots + u_5 y_{5j} \le v_1 x_{1j} + v_2 x_{2j} + \cdots + v_5 x_{5j} \quad (j = 1, \dots, 5) \\ v_1, v_{2,} \dots, v_5 \ge 0 \\ u_1, u_{2,} \dots, u_5 \ge 0 \end{cases} \tag{8}$$

in which x_{ij} is the average computation time of segmentation of the j-th category by using i-th algorithm (input) and y_{ij} is the average accuracy of segmentation of the j-th category by using i-th algorithm (output).By computing x_{ij} and y_{ij} then putting them in (8), we can obtain the efficiency of every DMU by solving (8). The results are provided in Table 4.

Method/Category	Animals	Buildings	Faces	Nature	Objects
Chan-Vese with $\mu = 0.01$	42.1287s	55.9337s	47.6588s	41.7300s	56.6916s
Chan-Vese with $\mu = 0.2$	42.3996s	52.8610s	50.5395s	39.1866s	64.7076s
Chan-Vese with $\mu = 0.4$	45.0279s	52.7354s	53.1786s	42.2463s	66.4899s
Chan-Vese with $\mu = 0.6$	44.6564s	51.2075s	64.2735s	44.3152s	67.6379s
Chan-Vese with $\mu = 0.8$	48.3813s	48.6384s	62.2584s	49.1314s	67.5419s
Chan-Vese with $\mu = 1$	53.6698s	52.1388s	54.9031s	51.8473s	67.5723s

Table 2. Average computation times for categories.

Chan-Vese with $\mu = 0.01$, $\mu = 0.2$ and $\mu = 0.8$ are efficient, all of these values can be selected for μ but the one which appears more than the other ones in the reference set of inefficient DMUs can be a better choice, because the presence of this

Method/Category	Animals	Buildings	Faces	Nature	Objects
Chan-Vese with $\mu = 0.01$	1462	1972	1723	2527	2625
Chan-Vese with $\mu = 0.2$	1401	2195	1757	4235	2246
Chan-Vese with $\mu = 0.4$	1587	2319	1855	4765	2332
Chan-Vese with $\mu = 0.6$	1881	2743	1889	4899	2344
Chan-Vese with $\mu = 0.8$	2175	3766	1910	4922	2407
Chan-Vese with $\mu = 1$	2237	3913	1849	5162	2787

Table 3. Average accuracy (MSSD) for every category.

Method	CCR efficiency
Chan-Vese with $\mu = 0.01$	1.0000
Chan-Vese with $\mu = 0.2$	1.0000
Chan-Vese with $\mu = 0.4$	0.9648
Chan-Vese with $\mu = 0.6$	0.9842
Chan-Vese with $\mu = 0.8$	1.0000
Chan-Vese with $\mu = 1$	0.9634

Table 4. Efficiency of methods.

Method	Reference set
Chan-Vese with $\mu = 0.4$	Chan-Vese with $\mu = 0.2$ and Chan-Vese with $\mu = 0.8$
Chan-Vese with $\mu = 0.6$	Chan-Vese with $\mu = 0.2$ and Chan-Vese with $\mu = 0.8$
Chan-Vese with $\mu = 1$	Chan-Vese with $\mu = 0.2$

Table 5. Reference set for inefficient DMUs.

parameter causes more number of methods to be inefficient. It is obvious from the Table 5 that we should choose 0.2 *as the most appropriate value for* μ.

Example 2 *In this example, similar to the previous one, the goal is to know what value of h (a value which represents smoothness of the evolving contour) among 1,2,3 and 4, makes Bernard method the most efficient. We do the same procedure by computing how different Bernard methods perform on each of categories and then we use the center of every cluster (category) as an input and an output (see Table 6 and 7) for the proposed DEA model (8). After solving the optimization problem for each DEA, we have Table 8.*

Method/Category	Animals	Buildings	Faces	Nature	Objects
Bernard with $h = 1$	95.7417s	122.1563s	133.6571s	101.3189s	122.8833s
Bernard with $h = 2$	52.6740s	49.6221s	48.3326s	48.6027s	46.8617s
Bernard with $h = 3$	41.4365s	38.0190s	38.7606s	37.3402s	39.0221s
Bernard with $h = 4$	43.1294s	38.8083s	39.2178s	38.3552s	38.2661s

Table 6. Average computation times for categories.

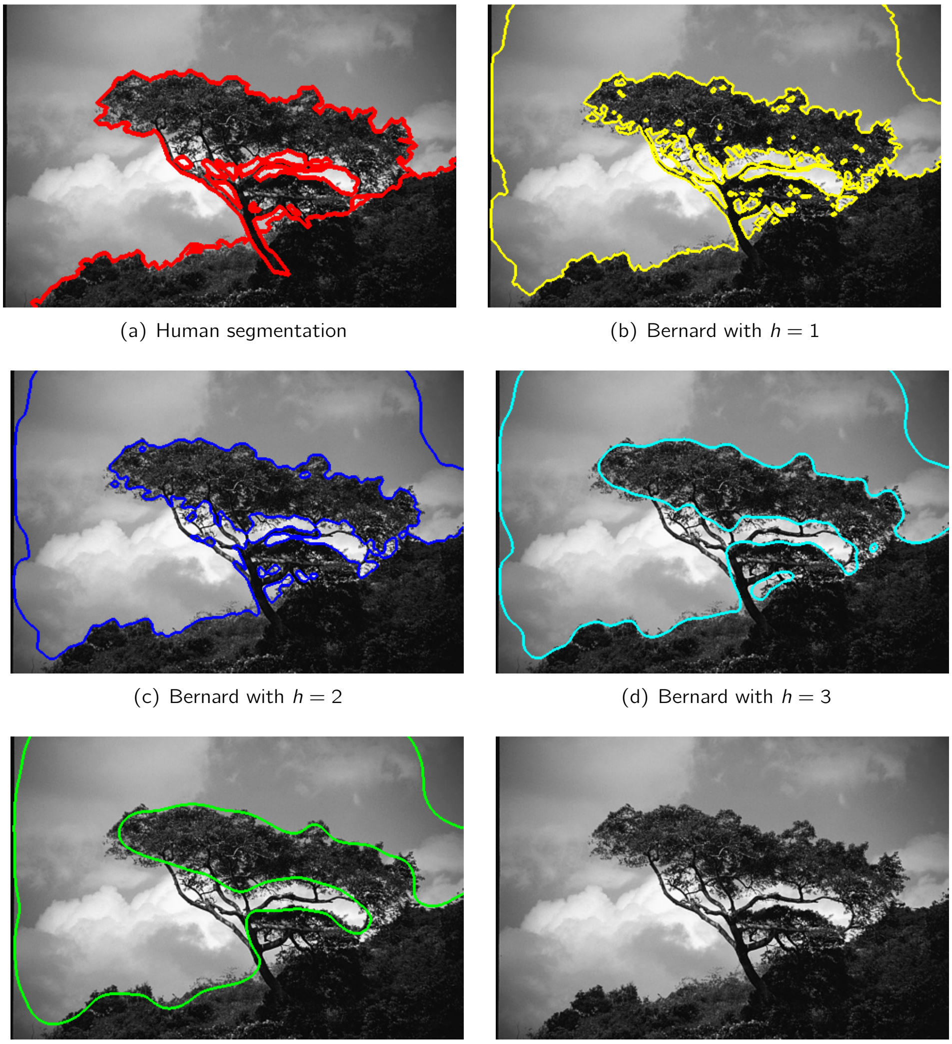
Method/Category	Animals	Buildings	Faces	Nature	Objects
Bernard with $h = 1$	1121	1765	1664	2015	2524
Bernard with $h = 2$	1003	1739	1601	2766	3615
Bernard with $h = 3$	1212	2445	1798	2936	4065
Bernard with $h = 4$	1421	3520	2656	3092	3935

Table 7. Average accuracy (MSSD) for every category.

As it can be seen, $h = 2$, $h = 3$ *and* $h = 4$ *are efficient, for choosing one of them as the best value for h, we check the reference sets of inefficient DEAs (Table 8). Thus the best option is* $h = 3$. *While* $h = 1$ *suggests a similar amount of accuracy to* $h = 2$ *case, high consumption of time and detection of small objects cause it to be inefficient (see for example Figure 2).*

Method	CCR efficiency	Reference set
Bernard with $h = 1$	0.5959	Bernard with $h = 3$
Bernard with $h = 2$	1.0000	-
Bernard with $h = 3$	1.0000	-
Bernard with $h = 4$	1.0000	-

Table 8. Efficiency of Bernard method with different h.



(a) Human segmentation

(b) Bernard with $h = 1$

(c) Bernard with $h = 2$

(d) Bernard with $h = 3$

(e) Bernard with $h = 4$

(f) Original picture

Figure 2. An example of how Bernard method with four different parameters performs for a picture of nature category. MSSD accuracy for segmentations (b), (c), (d) and (e) respectively are 12073.64, 124.37 and 104.92, computation times are 61.577s, 38.429s and 37.758s. MSSD computed with regard to the human segmentation (a). Take note again that the inverse of MSSD value is given to the DEA model for outputs.

Remark 4 *As it stated before, DEA computes the efficiency in the best situation for DEA while statistic methods consider the average performance. The proposed model in this paper, in some way, combines these two properties, it computes the average performance for each category and then sees if DMUs can be efficient by having these average performances as inputs and outputs. In this way, the proposed model deals with the problem of having excessive number of inputs and outputs.*

3. Conclusions

In this paper, a DEA-based model is proposed for comparing different image segmentation methods and also it can be used for a specific method with different parameters. The contribution of this paper is that instead of using the computed time and accuracy for an image as a separate input and output, the mean of computed times and accuracies of a class of images (for example animals) is used as an input and output.

It has been stated that if we want to perform methods on a large number of images then these images can be divided into some categories and the center of categories are inputs and outputs for the DEA model. In the end, some examples of finding an efficient parameter for an image segmentation method and finding efficient method(s) among a set of image segmentation algorithms by the proposed model are given.

Acknowledgement

The author is very thankful to the editor and reviewer for their time and valuable comments that improved the quality of this paper.

References

[1] D.J. Withey and Z.J. Koles. A Review of Medical Image Segmentation: Methods and Available Software, *International Journal of Bioelectromagnetism*, 10:125-148, 2008.

[2] I.L Lee, K. Lee and C. Torpelund-Bruin. *Journal of Computers*, V4, N11, 2009.

[3] M. J. M. Vasconcelos and J. M. R. S. Tavares. Image segmentation for human motion analysis: methods and applications, *8th. World Congress on Computational Mechanics*, 2008.

[4] N. Ikonomakis, K. N. Plataniotis and A. N. Venetsanopoulos. Color Image Segmentation for Multimedia Applications, *Journal of Intelligent and Robotic Systems*, 28:5-20, 2000.

[5] S. Samoilenko, K.-M. Osei-Bryson. Using Data Envelopment Analysis (DEA) for monitoring efficiency-based performance of productivity-driven organizations: Design and implementation of a decision support system, *Omega*, 41:131-142, 2013.

[6] Z. Zhou, G. Xu, C. Wang, and J. Wu. Modeling undesirable output with a DEA approach based on an exponential transformation: An application to measure the energy efficiency of Chinese industry, *Journal of Cleaner Production* 236:117717, 2019.

[7] V. Bosetti, M. Cassinelli and A. Lanza. Using Data Envelopment Analysis to Evaluate Environmentally Conscious Tourism Management. *FEEM Working Paper*, 59.04, 2004.

[8] M. Khodabakhshi, and K. Aryavash. The fair allocation of common fixed cost or revenue using DEA concept. Annals of Operations Research, 214(1):187-194, 2014.

[9] V. Charles and M. Kumar. *Data Envelopment Analysis and Its Applications to Management*, Cambridge Scholars Publishing, Newcastle, UK, 2012.

[10] J. Benneyan, M. E. Ceyhan and A. Sunnetci. Data envelopment analysis of national healthcare systems and their relative efficiencies, *Proceedings of the 37th International Conference on Computers and Industrial Engineering, Alexandria, Egypt*, 251-261, 2007.

[11] N. Tian, S. Tang, A. Che and P. Wu. Measuring regional transport sustainability using super-efficiency SBM-DEA with weighting preference, *Journal of Cleaner Production*, 242:118474, 2020.

[12] H.-E. Chueh and J.-Y. Jheng. Applying data envelopment analysis to evaluation of taiwanese solar cell industry operational performance, *International Journal of Computer Science & Information Technology (IJCSIT)*, 4 (4):1-8, 2012.

[13] K. Zhong, Y. Wang, J. Pei, S. Tang and Z. Han. (2021). Super efficiency SBM-DEA and neural network for performance evaluation. *Information Processing & Management*, 58(6):102728, 2021.

[14] M. Khodabakhshi. An output oriented super-efficiency measure in stochastic data envelopment analysis: Considering Iranian electricity distribution companies. *Computers & Industrial Engineering*, 58(4):663-671, 2010.

[15] M. Khodabakhshi. A one-model approach based on relaxed combinations of inputs for evaluating input congestion in DEA. *Journal of Computational and Applied Mathematics*, 230(2):443-450, 2009

[16] M. Khodabakhshi, and K. Aryavash. The cross-efficiency in the optimisticpessimistic framework. *Operational Research*, 17(2):619-632, 2017

[17] T. Chan and L. Vese. Active contours without edges. *IEEE Transactions on Image Processing,* 10(2):266-277, 2001.

[18] O. Bernard, D. Friboulet, P. Thevenaz, and M. Unser. Variational B-Spline Level-Set: A Linear Filtering Approach for Fast Deformable Model Evolution, *IEEE Trans. Image Process*, 18:1179-1191, 2009.

[19] W. W. Cooper, L. M. Seiford, K. Tone. *Data Envelopment Analysis A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, 2nd Edition, Springer New York, 2007.

[20] P. Arbelaez, M. Maire, C. Fowlkes, and J. Malik. Contour detection and hierarchical image segmentation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 33:898-916, 2011.

[21] T. Dietenbeck, M. Alessandrini, D. Friboulet, O. Bernard. CREASEG: a free software for the evaluation of image segmentation algorithms based on level-set. *IEEE International Conference On Image Processing*, Hong Kong, China, 2010.

[22] A. Hatami-Marbini, and M. Toloo. An extended multiple criteria data envelopment analysis model. *Expert Systems with Applications*, 73:201-219, 2017

