Received 18 January 2022
Accepted 11 February 2022

DOI: 10.52547/CMCMA.1.1.56
AMS Subject Classification: 65J20; 49M15; 74G20; 41A25

Ball comparison between three fourth convergence order schemes for nonlinear equations

Samundra Regmi[a], Ioannis K. Argyros[b], Santhosh George[c] and Christopher I. Argyros[d]

A ball convergence comparison is developed between three Banach space valued schemes of fourth convergence order to solve nonlinear models under $\omega-$continuity conditions on the derivative.

Keywords: Fourth convergence order scheme; Banach space; Nonlinear model.

1. Introduction

In this paper, we compare the ball convergence of three fourth convergence order method for solving equation

$$F(x) = 0. \tag{1.1}$$

Here $F : D \subset X \longrightarrow Y$ is continuously Fréchet differentiable, X, Y are Banach spaces, and D is a nonempty convex set.

Precisely, we consider the following three schemes:

Two step Newton scheme [1, 17]:

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= y_n - F'(y_n)^{-1}F(y_n). \end{aligned} \tag{1.2}$$

Jarratt type scheme [15]:

$$\begin{aligned} y_n &= x_n - \frac{2}{3}F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= x_n - \frac{1}{2}(3F'(y_n) - F'(x_n))^{-1}(3F'(y_n) + F'(x_n))F'(x_n)^{-1}F(x_n). \end{aligned} \tag{1.3}$$

Ostrowski-type scheme [16]:

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= y_n - A_n^{-1}F(y_n), \\ A_n &= 2[y_n, x_n; F] - F'(x_n), \end{aligned} \tag{1.4}$$

where $[.,.;F] : D \times D \longrightarrow L(X, Y)$ is a divided difference of order one.

[a] Learning Commons, University of North Texas at Dallas, Dallas, TX, USA. Email: samundra.regmi@untdallas.edu.
[b] Department of Mathematical Sciences, Cameron University, Lawton, OK 73505, USA. Email: iargyros@cameron.edu.
[c] Department of Mathematical and Computational Sciences,National Institute of Technology Karnataka, India-575 025. Email: sgeorge@nitk.edu.in.
[d] Department of Computing and Technology, Cameron University, Lawton, OK 73505, USA. Email: christopher.argyros@cameron.edu.
*Correspondence to: I.K. Argyros.

Earlier papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] have used conditions on derivatives up to the order five (not appearing on these methods) to prove the order of convergence. These conditions limit the applicability of the methods.

For example: Let $X = Y = \mathbb{R}$, $D = [-\frac{1}{2}, \frac{3}{2}]$. Define f on D by

$$f(t) = \begin{cases} t^3 \log t^2 + t^5 - t^4 & if\ \ t \neq 0 \\ 0 & if\ \ t = 0. \end{cases}$$

Then, we have $t_* = 1$,

$$f'''(t) = 6 \log t^2 + 60t^2 - 24t + 22.$$

Obviously $f'''(t)$ is not bounded on D. So, the convergence of these methods is not guaranteed by the analysis in earlier papers. Note that, we have used conditions only first derivative in the convergence analysis (notice that this is the only derivative appearing on the methods). We also provide a computable radius of convergence not given in earlier works [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. This way we locate a set of initial points for the convergence of the method. The numerical examples are chosen to show how the radii theoretically predicted are computed.

The article contains local convergence analysis in Section 2 and the numerical examples in Section 3.

2. Ball Convergence

We develop the convergence of schemes (1.2)-(1.4), respectively. Set $E = [0, \infty)$.

Assume there exists function $\omega_0 : E \longrightarrow E$ which is nondecreasing and continuous such that equation

$$\omega_0(t) - 1 = 0 \tag{2.1}$$

has a minimal solution $R \in E - \{0\}$. Set $E_0 = [0, R)$.

Assume there exists function where $\omega : E_0 \longrightarrow E$ which is continuous and nondecreasing such that equation

$$g_1(t) - 1 = 0 \tag{2.2}$$

has a minimal solution $r_1 \in (0, R)$, where

$$g_1(t) = \frac{\int_0^1 \omega((1-\tau)t)d\tau}{1 - \omega_0(t)}.$$

Assume equation

$$\omega_0(g_1(t)t) - 1 = 0 \tag{2.3}$$

has a minimal solution $R_1 \in (0, R)$.

Assume equation

$$g_2(t) - 1 = 0 \tag{2.4}$$

has a minimal solution $r_2 \in (0, R_1)$, where

$$g_2(t) \quad = \quad g_1(g_1(t)t))g_1(t).$$

We shall prove that

$$r = \min\{r_i\}, \ \ i = 1, 2, \tag{2.5}$$

is a radius of convergence for scheme (1.2). The definition of r gives

$$0 \leq \omega_0(t) < 1, \tag{2.6}$$

$$0 \leq \omega_0(g_1(t)t) < 1, \tag{2.7}$$

and

$$0 \leq g_i(t) < 1 \tag{2.8}$$

for all $t \in [0, r)$.

Let $T(x, a), \bar{T}(x, a)$ denote the open and closed balls, respectively in X with center $x \in X$ and of radius $a > 0$.

The conditions (A) to be used in the convergence are:

(a1) $F : D \subset X \longrightarrow Y$ is Fréchet continuously differentiable and $x^* \in D$ is a simple solution of equation $F(x) = 0$.

(a2) There exists a continuous and nondecreasing function $\omega_0 : E \longrightarrow E$ such that for each $x \in D$

$$\|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \leq \omega_0(\|x - x^*\|).$$

Set $D_0 = D \cap T(x^*, R)$.

(a3) There continuous and nondecreasing exist function $\omega : E_0 \longrightarrow E$ such that for each $x, y \in D_0$

$$\|F'(x^*)^{-1}(F'(x) - F'(y))\| \leq \omega(\|x - y\|).$$

(a4) $\bar{T}(x^*, r) \subset D$.
and

(a5) There exists $r_* \geq r$ such that $\int_0^1 \omega_0(\tau r_*)d\tau < 1$. Set $D_1 = D \cap \bar{T}(x^*, r_*)$.

Next, we develop the ball convergence result for method (1.2) based on conditions (A) and the preceding notation.

THEOREM 2.1 *Under conditions (A) further choose $x_0 \in T(x^*, r) - \{x^*\}$. Then, sequence $\{x_n\}$ generated by method (1.2) is well defined in $T(x^*, r)$, remains in $T(x^*, r)$ for all $n = 0, 1, 2, \dots,$ and converges to x^* such that*

$$\|y_n - x^*\| \leq g_1(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\| < r, \tag{2.9}$$

and

$$\|x^{n+1} - x^*\| \leq g_2(\|x_n - x^*\|\|x_n - x^*\| \leq \|x_n - x^*\|, \tag{2.10}$$

with the functions $g_i, i = 1, 2$ defined previously and radius r given in (2.5). Moreover, x^* is the only solution of equation $F(x) = 0$ in the set D_1 given in (a5).

Proof. Mathematical induction on integer j will validate estimates (2.9) and (2.10). Choose arbitrary $u \in T(x^*, r) - \{x^*\}$. Then, using (2.5), (2.6), (a1) and (a2), we get in turn

$$\|F'(x^*)^{-1}(F'(u) - F'(x^*))\| \leq \omega_0(\|u - x^*\|) \leq \omega_0(r) < 1, \tag{2.11}$$

implying together with a Lemma on invertible operators due to Banach [8] that $F'(u)^{-1} \in L(Y, X)$, and

$$\|F'(u)^{-1}F'(x^*)\| \leq \frac{1}{1 - \omega_0(\|u - x^*\|)}. \tag{2.12}$$

So y_0 are well defined too by the first substep of method (1.2) for $j = 0$. We can write

$$\begin{aligned} y_0 - x^* &= x_0 - x^* - F'(x_0)^{-1}F(x_0) \\ &= [F'(x_0)^{-1}F'(x^*)] \times [\int_0^1 F'(x^*)^{-1}(F'(x^* + \tau(x_0 - x^*)) - F'(x_0))d\tau(x_0 - x^*)]. \end{aligned} \tag{2.13}$$

Using (2.5), (2.8) (for $i = 1$), (a1)-(a3), (2.12) (for $u = x_0$) and (2.13), we have in turn

$$\|y_0 - x^*\| \leq \frac{\int_0^1 \omega((1 - \tau)\|x_0 - x^*\|)d\tau\|x_0 - x^*\|}{1 - \omega_0(\|x_0 - x^*\|)} \leq g_1(\|x_0 - x^*\|)\|x_0 - x^*\| \leq \|x_0 - x^*\| < r, \tag{2.14}$$

proving $y_0 \in T(x^*, r)$, and (2.9) holds for $j = 0$. Then, (2.12) also holds for $u = y_0$, and x_1 is well defined by the second substep of scheme (1.2) for $j = 0$. Moreover, we can write

$$x_1 - x^* = y_0 - x^* - F'(y_0)^{-1}F(y_0). \tag{2.15}$$

Then, as in (2.13), but using (2.8) (for $j = 2$), (2.14) and (2.15), we get in turn

$$\begin{aligned} \|x_1 - x^*\| &\leq g_1(\|y_0 - x^*\|))\|y_0 - x^*\| \\ &\leq g_1(g_1(\|x_0 - x^*\|)\|x_0 - x^*\|)g_1(\|x_0 - x^*\|)\|x_0 - x^*\| \\ &\leq g_2(\|x_0 - x^*\|)\|x_0 - x^*\| \leq \|x_0 - x^*\|, \end{aligned} \tag{2.16}$$

proving $x_1 \in T(x^*, r)$and (2.10) hold for $j = 0$.

Furthermore, replace x_0, y_0, x^1 by x_n, y_n, x_{n+1} in the previous calculations to complete the induction for items (2.9) and (2.10). Then in view of the estimation

$$\|x_{n+1} - x^*\| \leq d\|x_n - x^*\| < r, \tag{2.17}$$

where $d = g_3(\|x_0 - x^*\|) \in [0, 1)$, we conclude $\lim_{n\longrightarrow\infty} x_n = x^*$ and $x_{n+1} \in T(x^*, r)$.

Finally, we show the uniqueness of the solution by considering $y^* \in D_1$ with $F(y^*) = 0$. Then by (a1), (a2) and (a5), we obtain in turn

$$\|F'(x^*)^{-1}(G - F'(x^*))\| \leq \int_0^1 \omega_0(\tau\|x^* - y^*)\|d\tau \leq \int_0^1 \omega_0(\tau r_*)d\tau < 1,$$

for $G = \int_0^1 F'(x^* + \tau(x^{**} - x^*))d\tau$. Hence, $G^{-1} \in L(Y, X)$. Consequently, from $0 = F(y^*) - F(x^*) = G(y^* - x^*)$, we obtain $y^* = x^*$.

□

Next, we develop the ball convergence of scheme (1.3) along the same lines. We add the condition

(a3)' There exists function $\omega_1 : E_0 \longrightarrow E$ continuous and nondecreasing such that for each $x \in D_0$

$$\|F'(x^*)^{-1}F'(x)\| \leq \omega_1(\|x - x^*\|).$$

The "g" functions are

$$\bar{g}_2(t) = g_1(t) + \frac{\int_0^1 \omega_1(\tau t)d\tau}{1 - \omega_0(t)}$$

and

$$\bar{g}_2(t) = g_1(t) + \frac{3(\omega_0(t) + \omega_0(\bar{g}_1(t)t)) \int_0^1 \omega_1(\tau t)d\tau}{4(1 - \omega_0(t))(1 - p(t))},$$

where

$$p(t) = \frac{1}{2}(3\omega_0(\bar{g}_1(t)t) + \omega_0(t)).$$

Denote by $\bar{r}_1, \bar{r}_2$ the minimal positive solutions of equations (if they exist)

$$\bar{g}_1(t) - 1 = 0 \tag{2.18}$$

and

$$\bar{g}_2(t) - 1 = 0. \tag{2.19}$$

Set

$$\bar{r} = \min\{\bar{r}_i\}, i = 1, 2. \tag{2.20}$$

Denote by (A') condition (A) and (a3)'. Then, using scheme (1.3) and estimates

$$y_n - x^* = x_n - x^* - F'(x_n)^{-1}F(x_n) + \frac{1}{3}F'(x_n)^{-1}F(x_n),$$

$$\|y_n - x^*\| \leq [g_1(\|x_n - x^*\|) + \frac{\int_0^1 \omega_1(\tau\|x_n - x^*\|)d\tau}{1 - \omega_0(\|x_n - x^*\|)}]\|x_n - x^*\| \leq \bar{g}_1(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\| < \bar{r},$$

$$\begin{aligned}
\|(2F'(x^*))^{-1}(3F'(y_n) - F'(x_n) - 2F'(x^*))\| &\leq \frac{1}{2}(3\|F'(x^*)^{-1}(F'(y_n) - F'(x^*))\| + \|F'(x^*)^{-1}(F'(x_n) - F'(x^*))\| \\
&\leq p(\|x_n - x^*\| < p(\bar{r}) < 1,
\end{aligned}$$

$$\|(3F'(y_n) - F'(x_n) - 2F'(x^*))^{-1}F'(x^*)\| \leq \frac{1}{2(1 - p(\|x_n - x^*\|))},$$

$$\begin{aligned}
x_{n+1} - x^* &= x_n - x^* - F'(x_n)^{-1}F(x_n) + [I - \frac{1}{2}(3F'(y_n) - F'(x_n))^{-1}(3F'(y_n) + F'(x_n))]F'(x_n)^{-1}F(x_n) \\
&= x_n - x^* - F'(x_n)^{-1}F(x_n) - \frac{3}{2}(3F'(y_n) - F'(x_n))^{-1}(F'(y_n) - F'(x_n))F'(x_n)^{-1}F(x_n),
\end{aligned}$$

so

$$\begin{aligned}
\|x_{n+1} - x^*\| &\leq [g_1(\|x_n - x^*\|) + \frac{3(\omega_0(\|x_n - x^*\|) + \omega_0(\|y_n - x^*\|)) \int_0^1 \omega_1(\tau\|x_n - x^*\|)d\tau}{4(1 - p(\|x_n - x^*\|))(1 - \omega_0(\|x_n - x^*\|))}]\|x_n - x^*\| \\
&\leq \bar{g}_2(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\|.
\end{aligned}$$

Hence, we arrive at the corresponding ball convergence result for scheme (1.3).

THEOREM 2.2 *Under conditions (A) further assume $x_0 \in T(x^*, \bar{r}) - \{x^*\}$. Then, the conclusions of Theorem 2.1 but for scheme (1.3) hold with $\bar{g}_1, \bar{g}_2, \bar{r}$ replacing g_1, g_2, r, respectively.*

Next, we present the ball convergence of scheme (1.4). We add the condition

(a3)'' There exists function $\omega_2 : E_0 \times E_0 \longrightarrow E$ continuous and nondecreasing such that for each $x, y \in D_0$

$$\|F'(x^*)^{-1}([y, x; F] - F'(x^*))\| \leq \omega_2(\|y - x^*\|, \|x - x^*\|),$$

provided that $[., .; F] : D_0 \times D_0 \longrightarrow L(X, Y)$. Denote by (A'') conditions (A), (a3)' and (a3)''. Define functions $\bar{\bar{g}}_2$ and q by

$$\bar{\bar{g}}_2(t) = g_1(g_1(t)) + \frac{(2\omega_2(g_1(t)t, t) + \omega_0(t) + \omega_0(g_1(t)t)) \int_0^1 \omega_1(\tau g_1(t))d\tau}{(1 - \omega_0(g_1(t)t)(1 - q(t))}$$

and

$$q(t) = 2\omega_2(g_1(t)t, t) + \omega_0(t)).$$

Denote by $\bar{\bar{r}}_2$ the minimal positive solutions of equations

$$\bar{\bar{g}}_2(t) - 1 = 0 \tag{2.21}$$

and set

$$\bar{\bar{r}} = \min\{r_1, \bar{\bar{r}}_2\}. \tag{2.22}$$

We also use the estimate

$$\begin{aligned}
\|x_{n+1} - x^*\| &= \|y_n - x^* - F'(y_n)^{-1}F(y_n) + (F'(y_n)^{-1} - (2[y_n, x_n; F] - F'(x_n))^{-1})F(y_n)\| \\
&= \|y_n - x^* - F'(y_n)^{-1}F(y_n) + F'(y_n)^{-1}(2([y_n, x_n; F] - F'(x^*)) + (F'(x^*) - F'(x_n)) \\
&\quad +(F'(x^*) - F'(y_n))(2[y_n, x_n; F] - F'(x_n))^{-1}F(y_n)\| \\
&\leq [g_1(\|y_n - x^*\|) \\
&\quad + \frac{(2\omega_2(\|y_n - x^*\|, \|x_n - x^*\|) + \omega_0(\|x_n - x^*\|) + \omega_0(\|y_n - x^*\|)) \int_0^1 \omega_1(\tau\|y_n - x^*\|)d\tau}{(1 - \omega_0(\|y_n - x^*\|))(1 - q(\|x_n - x^*\|))}]\|y_n - x^*\| \\
&\leq \bar{\bar{g}}_2(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\| < \bar{r},
\end{aligned}$$

where we also used

$$\begin{aligned}
\|F'(x^*)^{-1}(2[y_n, x_n; F] - F'(x_n)) - F'(x^*))\| &\leq 2\|F'(x^*)^{-1}([y_n, x_n; F] - F'(x^*))\| + \|F'(x^*)^{-1}(F'(x_n) - F'(x^*))\| \\
&\leq 2\omega_2(g_1(\|x_n - x^*\|)\|x_n - x^*\|, \|x_n - x^*\|) + \omega_0(\|x_n - x^*\|)) \\
&= q(\|x_n - x^*\|) \leq q(\bar{\bar{r}}) < 1,
\end{aligned}$$

so

$$\|(2[y_n, x_n; F] - F'(x^*))^{-1}F'(x^*)\| \leq \frac{1}{1 - q(\|x_n - x^*\|)}.$$

Hence, we arrive at the ball convergence result for scheme (1.4).

THEOREM 2.3 *Under condition (A'') further assume $x_0 \in T(x^*, \bar{\bar{r}}) - \{x^*\}$. Then, the conclusions of Theorem 2.1 but for scheme (1.4) hold with $\bar{\bar{g}}_2, \bar{\bar{r}}$ replacing g_2, r, respectively.*

REMARK 2.4 1. *In view of (2.9) and the estimate*

$$\|F'(x^*)^{-1}F'(x)\| = \|F'(x^*)^{-1}(F'(x) - F'(x^*)) + I\| \leq 1 + \|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \leq 1 + L_0\|x - x^*\|$$

condition (2.12) can be dropped and ω_1 can be replaced by

$$\omega_1(t) = 1 + \omega_0(R)$$

or

$$\omega_1(t) = 1 + \omega_0(R),$$

since $t \in [0, R)$.

2. *The results obtained here can be used for operators F satisfying autonomous differential equations [2] of the form*

$$F'(x) = P(F(x))$$

where P is a continuous operator. Then, since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing x^*. For example, let $F(x) = e^x - 1$. Then, we can choose: $P(x) = x + 1$.

3. *Let* $\omega_0(t) = L_0 t$*, and* $\omega(t) = Lt$*. In [2, 3] we showed that* $r_A = \frac{2}{2L_0+L}$ *is the convergence radius of Newton's method:*

$$x^{n+1} = x_n - F'(x_n)^{-1}F(x_n) \text{ for each } n = 0, 1, 2, \cdots \tag{2.23}$$

under the conditions (a1) - (a3). It follows from the definition of r in (2.5) that the convergence radius r of the method (1.2) cannot be larger (same for $\bar{r}, \bar{\bar{r}}$*) than the convergence radius* r_A *of the second order Newton's method (2.23). As already noted in [2, 3]* r_A *is at least as large as the convergence radius given by Rheinboldt [18]*

$$r_R = \frac{2}{3L}, \tag{2.24}$$

where L_1 *is the Lipschitz constant on D. The same value for* r_R *was given by Traub [21]. In particular, for* $L_0 < L_1$ *we have that*

$$r_R < r_A$$

and

$$\frac{r_R}{r_A} \to \frac{1}{3} \text{ as } \frac{L_0}{L_1} \to 0.$$

That is the radius of convergence r_A *is at most three times larger than Rheinboldt's.*

4. *We can compute the computational order of convergence (COC) defined by*

$$\mu = \ln\left(\frac{\|x_{n+1} - x^*\|}{\|x_n - x^*\|}\right) / \ln\left(\frac{\|x_n - x^*\|}{\|x_{n-1} - x^*\|}\right)$$

or the approximate computational order of convergence

$$\mu_1 = \ln\left(\frac{\|x_{n+1} - x_n\|}{\|x_n - x_{n-1}\|}\right) / \ln\left(\frac{\|x_n - x_{n-1}\|}{\|x_{n-1} - x_{n-2}\|}\right).$$

The calculation of these parameters does not require the usage of higher than one derivatives.

3. Numerical Examples

EXAMPLE 3.1 *[9] Consider the kinematic system*

$$F_1'(x) = e^x,\ F_2'(y) = (e-1)y + 1,\ F_3'(z) = 1$$

with $F_1(0) = F_2(0) = F_3(0) = 0$*. Let* $F = (F_1, F_2, F_3)$*. Let* $X = Y = \mathbb{R}^3, D = \bar{U}(0, 1), x^* = (0, 0, 0)^T$*. Define function F on D for* $w = (x, y, z)^T$ *by*

$$F(w) = (e^x - 1, \frac{e-1}{2}y^2 + y, z)^T.$$

Then, we get

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

so $\omega_0(t) = (e-1)t, \omega(t) = e^{\frac{1}{e-1}}t, \omega_1(t) = e^{\frac{1}{e-1}}, \omega_2(s, t) = \frac{1}{2}(e-1)(s+t)$*. Then, the radii:*

$$r = 0.382692,\ \bar{r} = 0.067144,\ \bar{\bar{r}} = 0.301641.$$

EXAMPLE 3.2 *Consider* $X = Y = C[0, 1]$*,* $D = \overline{U}(0, 1)$ *and* $F : D \longrightarrow Y$ *defined by*

$$F(\phi)(x) = \varphi(x) - 5\int_0^1 x\tau\phi(\tau)^3 d\tau. \tag{3.1}$$

We have that

$$F'(\phi(\xi))(x) = \xi(x) - 15\int_0^1 x\tau\phi(\tau)^2\xi(\tau)d\tau, \text{ for each } \xi \in D.$$

Then, we get that $x^* = 0$*, so* $\omega_0(t) = 15t, \omega(t) = 30t$ *and* $\omega_1(t) = 30, \omega_2(s, t) = 7.5(s+t)$*. Then, the radii:*

$$r = 0.3333,\ \bar{r} = 0.00089380,\ \bar{\bar{r}} = 0.000823048.$$

EXAMPLE 3.3 *Let* $X = Y = \mathbb{R}$*,* $D = [-\frac{1}{2}, \frac{3}{2}]$*. Define F on D by*

$$F(t) = \begin{cases} t^3 \log t^2 + t^5 - t^4 & \text{if } t \neq 0 \\ 0 & \text{if } t = 0. \end{cases}$$

Then, we have $t_* = 1$*, We have* $\omega_0(t) = \omega(t) = 96.6629073t, \omega_1(t) = 2$ *and* $\omega_2(s, t) = \frac{96.6629073}{2}(s+t)$*. Then, the radii:*

$$r = 0.00689682,\ \bar{r} = 0.001006076,\ \bar{\bar{r}} = 0.000138102.$$

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